Baryons beyond the quark-diquark model

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with

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Andreas Krassnigg
Diana Nicmorus
(University of Graz, Austria)

Motivation

Hadron structure:

- **Nucleon & delta** properties: masses, em. form factors, N→$\Delta\gamma$ transition, . . .
- Are baryons predominantly **quark-diquark** systems?
- Chiral structure, role of the **pion cloud**
- How are quarks and gluons distributed in hadrons? How does the **proton spin** come about?

A comprehensive description of hadrons within QCD is needed.
Chiral symmetry breaking, dynamical mass generation, confinement

Covariant bound-state equations

- **Dyson-Schwinger equations** (DSEs) of QCD provide input: Green functions
- Ab-initio, but **truncations** necessary "QCD modeling"
- Chiral limit $\longleftrightarrow$ heavy quarks
- Complements lattice QCD, quark models, effective field theories.

Mesons:

Bethe-Salpeter equation (BSE)
Maris, Roberts '97; Maris, Tandy '99; ....

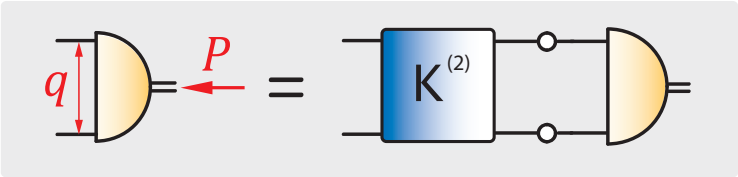
Baryons:

Quark-diquark model
Oettel, Hellstern, Alkofer, Reinhardt '98,
GE, Alkofer, Krassnigg, Schwinzerl '08, ...

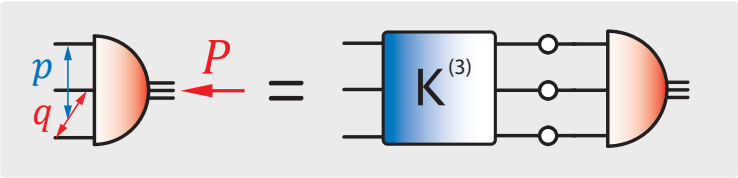
Faddeev equation
GE, Alkofer, Krassnigg, Nicmorus: PRL 104, 201601 (2010)

Bound-state equations

Mesons:
Bethe-Salpeter equation



Baryons:
Three-body equation



- **Bound-state amplitudes**
  On-shell: $P^2 = -M^2$

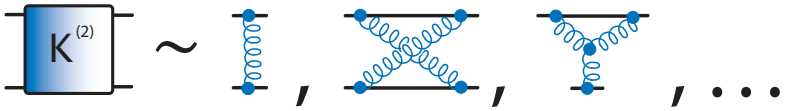
$$\Gamma(q,P) = \sum_i f_i(q^2,\ \hat{q}\cdot\hat{P}\,)\ \tau_i\,(q,P)$$

Covariants $\Leftrightarrow$ quark-spin and OAM eigenstates in particle's restframe:

- 0^- meson: 4 covariants: $\gamma^5, \ldots$ (s,p)
- 1^- meson: 8 covariants: $\gamma^\mu, \ldots$ (s,p,d)
- Nucleon: 64 covariants (s,p,d)
  (Quark-diquark model: 8)

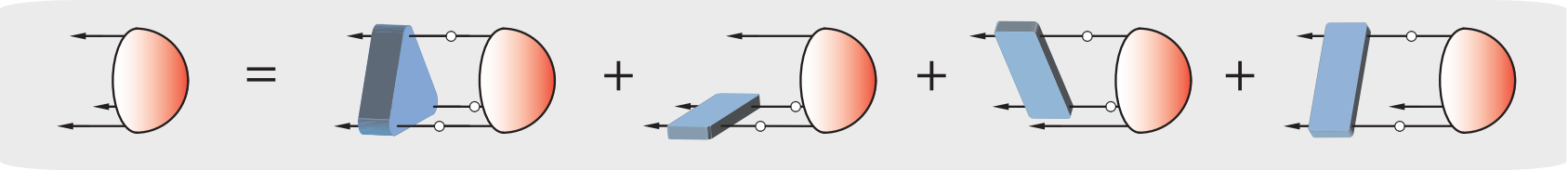
- **Dressed quark propagator**
  solve quark DSE $\Rightarrow$ $M(p^2), A(p^2)$.

- **2- and 3-quark kernels**
  the 'modeling' part.



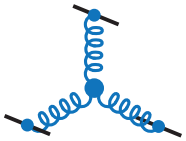
Bound-state equations

Three-body equation:



Irreducible 3-body diagrams

3-gluon coupling to each quark, ...

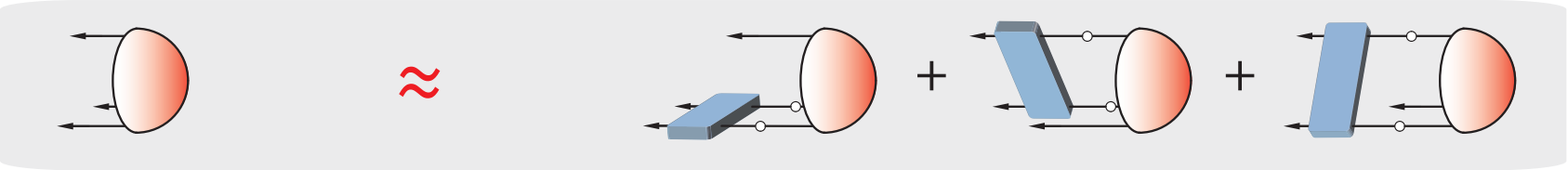


Quark-quark correlations

assumed as dominant structure in baryons.
Hints: lattice QCD, BSE, hadron spectrum, ...

Bound-state equations

Faddeev equation:



Quark-quark correlations

assumed as dominant structure in baryons.
Hints: lattice QCD, BSE, hadron spectrum, ...

Still numerically expensive,
but we can do this now.

Full covariant structure
of the amplitude!

GE, Alkofer, Krassnigg, Nicmorus:
PRL 104, 201601 (2010)
EPJ Web Conf. 3, 3028 (2010)

Need to know

- **Quark propagator**
  obtained from its Dyson-Schwinger equation
- **2-quark kernel**
  Practical strategy: construct ansätze
  which satisfy AXWTI (⇔ chiral symmetry)

DSEs

Dyson-Schwinger equations:

Quantum equations of motion for QCD's Green functions.

Infinitely coupled system.

Roberts, Williams: Prog.Part.Nucl.Phys. 33 (1994)

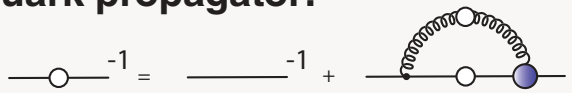
Alkofer, von Smekal: Phys.Rept. 353 (2001)

Fischer: J. Phys. G 32 (2006)

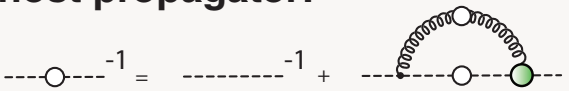
Momentum limits:
- UV: Perturbation theory
- IR: Infrared exponents

Numerical solutions require **truncations!**

Quark propagator:

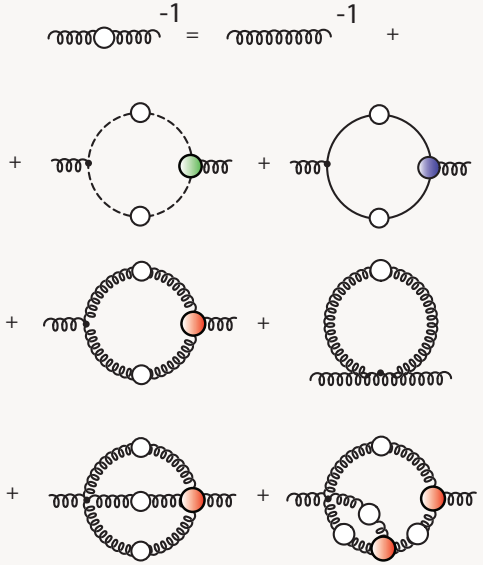


Ghost propagator:



Ghost-gluon vertex:

Gluon propagator:



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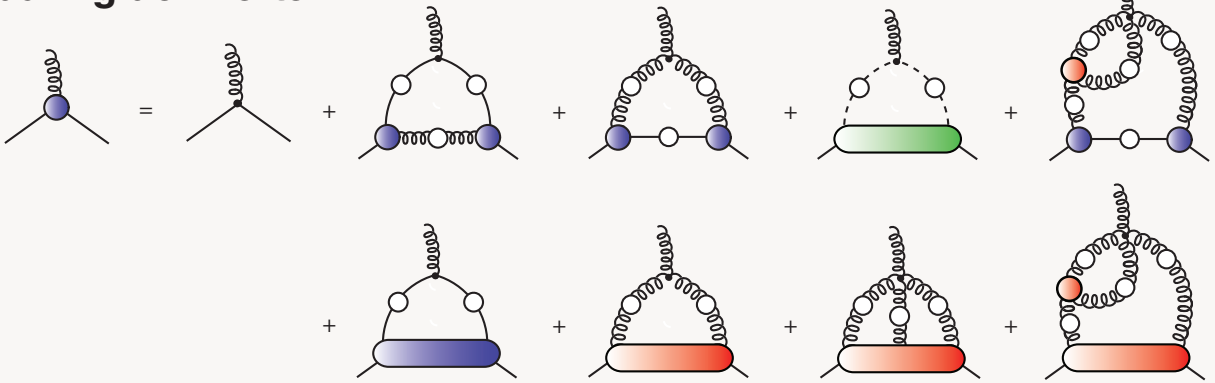

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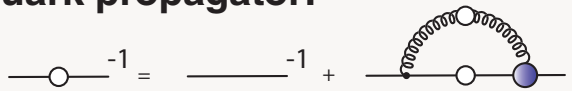

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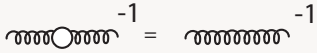
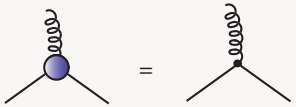


$D(k^2)^{-1}$

"Rainbow truncation"

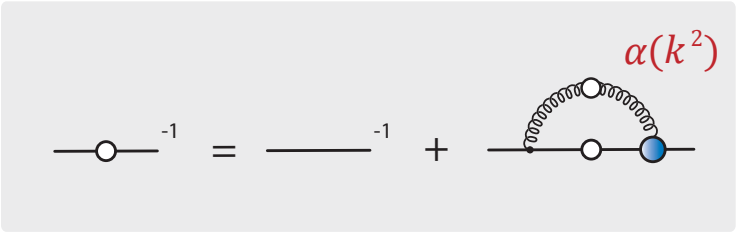
Quark-gluon vertex:



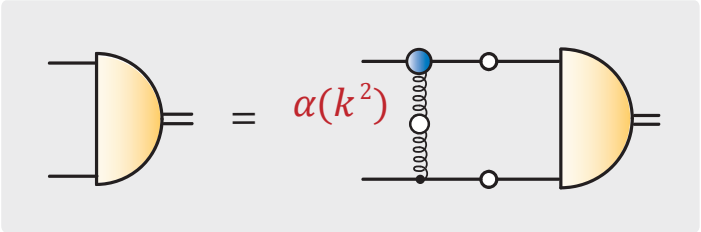
$\Gamma(k\ )$

Rainbow-Ladder Truncation

Quark DSE: "Rainbow"



Meson BSE: "Ladder"



Why rainbow-ladder?

- simple
- exact in the UV
- respects chiral symmetry & its spontaneous breaking

  ⇒ GMOR: $f_\pi^2 m_\pi^2 = 2m_q\langle q\bar{q}\rangle$
  chiral-limit pion is massless

Good description (up to the bottom quark) of pseudoscalar & vector-meson ground states

Effective coupling $\alpha(k^2)$ is model input.

Rainbow-Ladder Truncation

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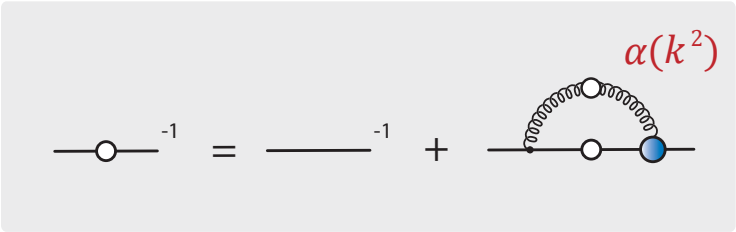

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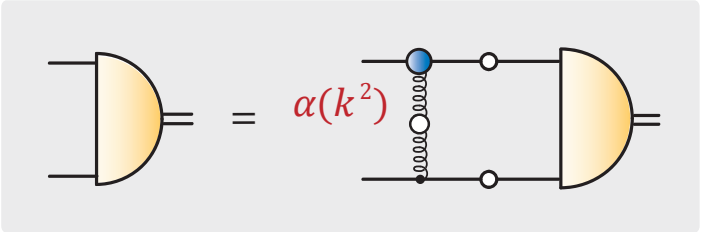

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⇒ see talks by
Christian Fischer & Richard Williams

What's beyond rainbow-ladder?

- **Pion cloud: Attractive** (~100 MeV) in chiral region; reduces hadron masses, decay constants; enhances charge radii

NJL: Oertel, Buballa, Wambach (2001)
BSE: Pichowsky, Walawalkar, Capstick (1999) ;
Fischer, Nickel, Wambach (2007); Fischer, Williams (2008)

- **Non-resonant corrections:**
  Repulsive (~100 MeV for m_ρ);
  Cancellation with pion cloud?
  Fischer, Williams, PRL 103 (2009)

Rainbow-Ladder Truncation

Effective coupling:
the only model input!

$$\alpha(k^2) = c\,\alpha_{\rm IR}(k^2,\omega) + \alpha_{\rm UV}(k^2)$$

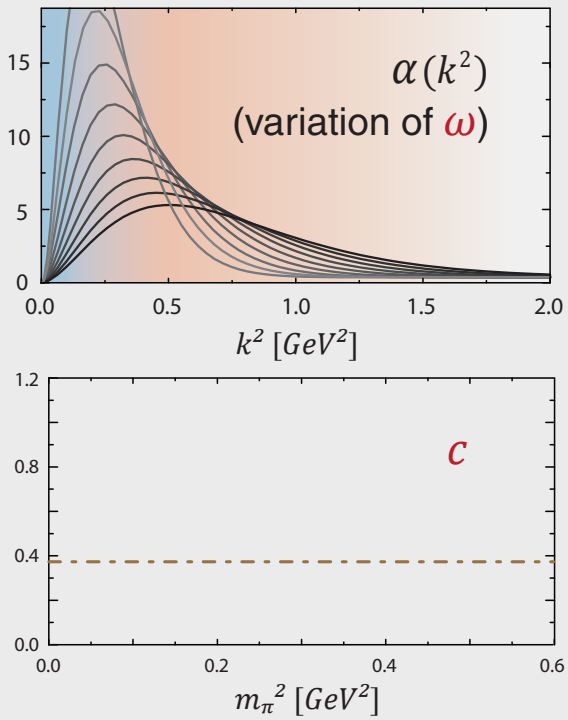
Maris, Tandy: PRC 60 (1999)

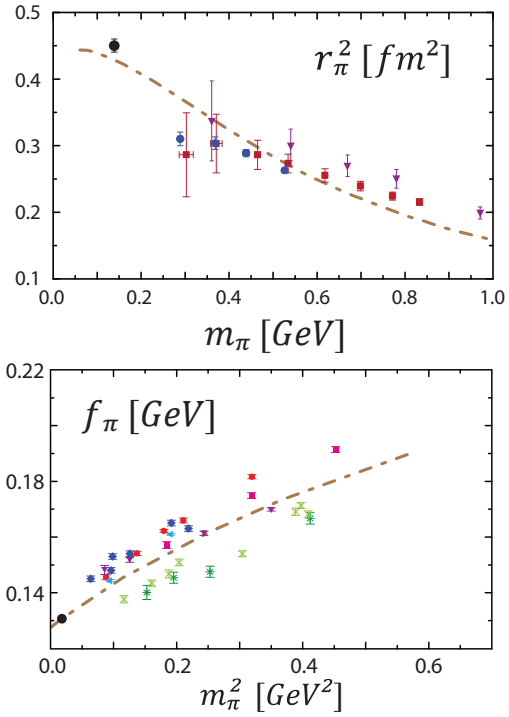
- **'Fixed strength'** (- - - - -): adjust c to reproduce **experiment**

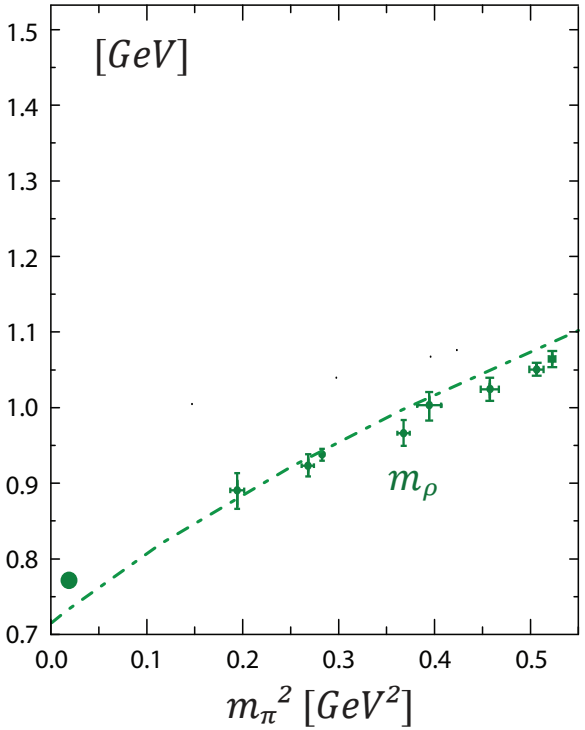
⇒ good description of π, ρ ground states

Maris, Roberts, Tandy, Bhagwat, Krassnigg, …

results **insensitive** to ω:
depend only on **"integrated strength"**







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GE, Alkofer, Cloet, Krassnigg, Roberts : PRC 77 (2008)

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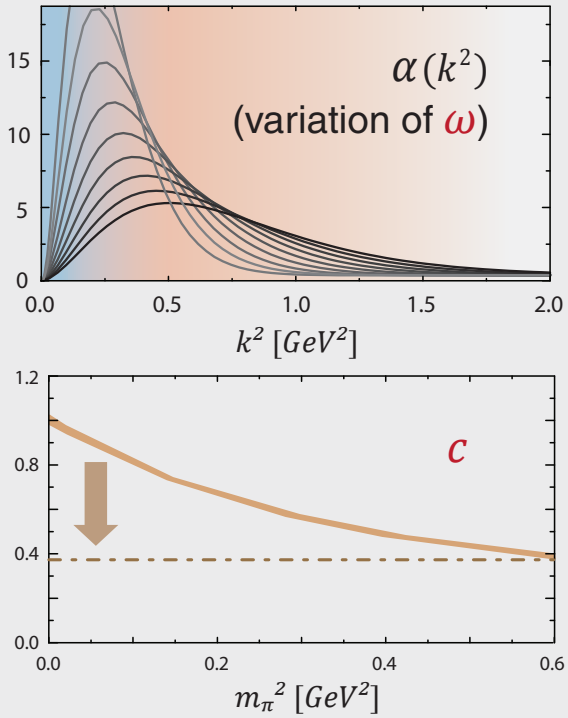
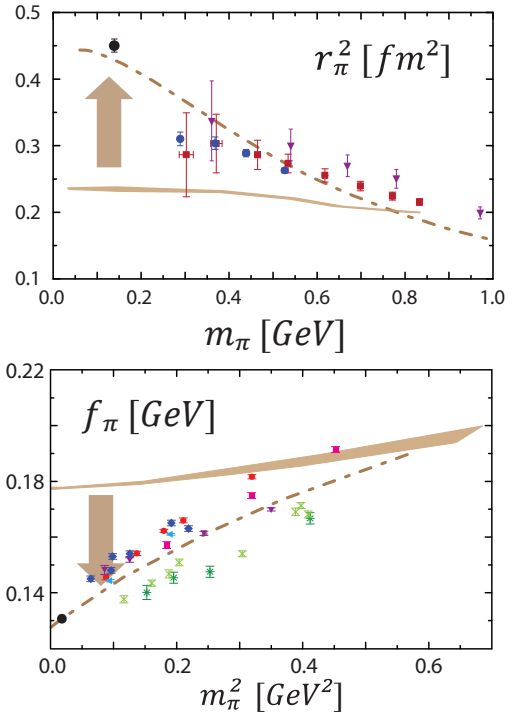
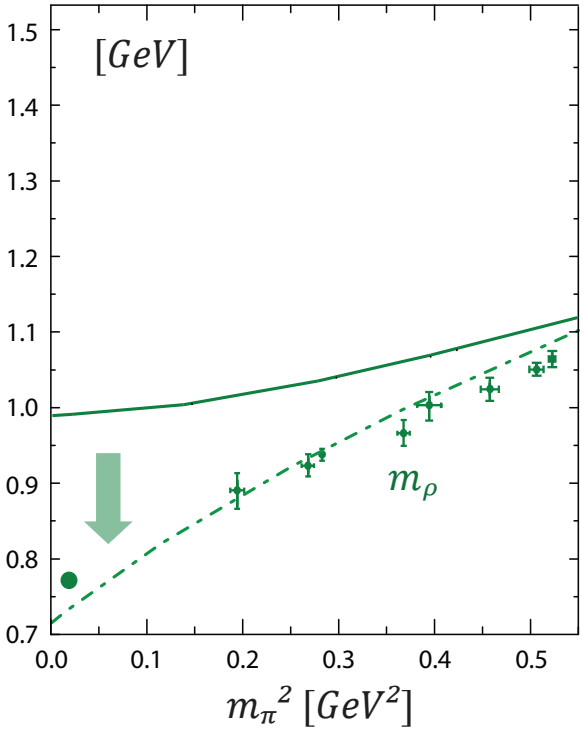
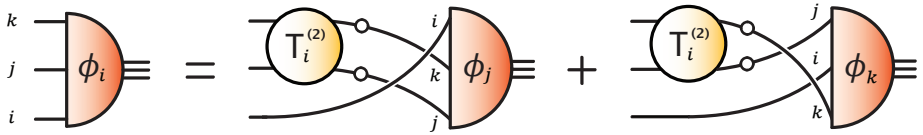




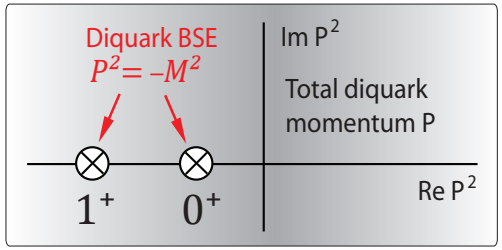


A quick detour: the quark-diquark model

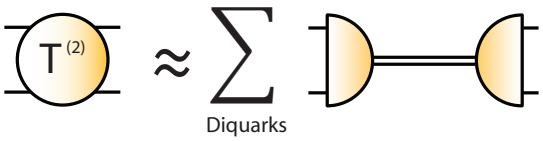
- **Faddeev equations**
  neglect 3q correlations: pairwise qq interaction;
  involve qq T-matrix as intermediate step



- **Rainbow-ladder kernel:**
  timelike diquark poles in T-matrix

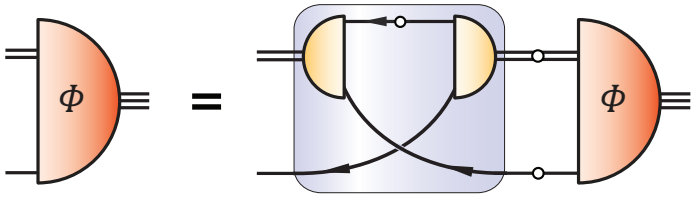


- **Diquark ansatz** for qq T-matrix:



- **Diquark BSE:** solve for lightest diquarks:
  scalar (0^+), **axial-vector** (1^+).
  ⇒ diquark masses & amplitudes

- **Quark-diquark BSE**
  ⇒ nucleon mass & amplitudes

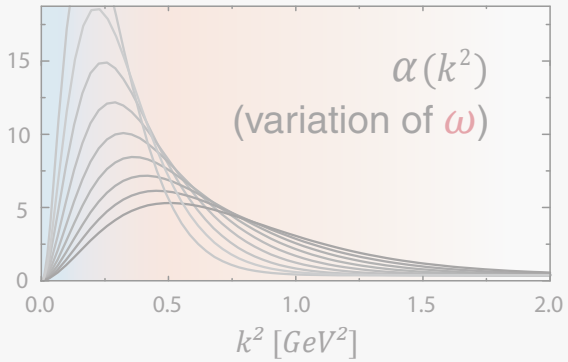


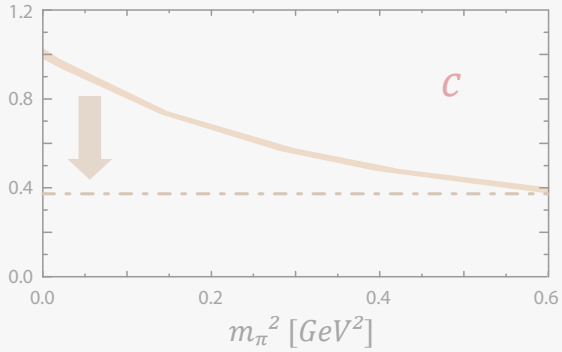
Quark exchange between quark and diquark binds nucleon; **gluon exchange** between quarks binds diquarks.

A quick detour: the quark-diquark model

Effective coupling:
the only model input!

$$\alpha(k^2) = c\,\alpha_{\rm IR}(k^2,\omega) + \alpha_{\rm UV}(k^2)$$



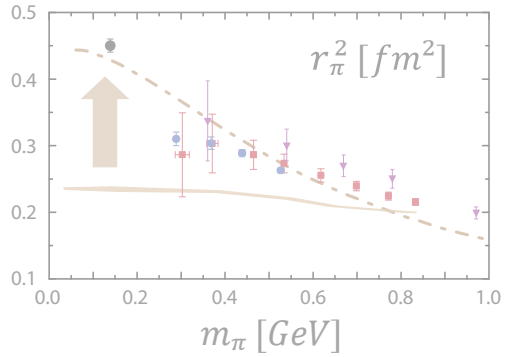


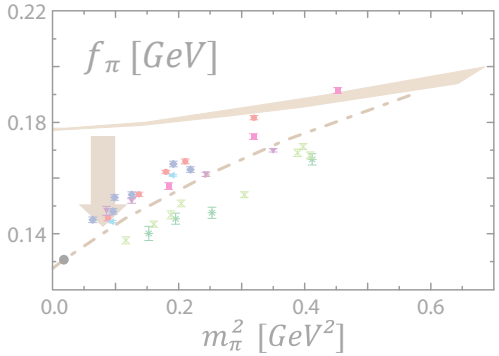
- **'Fixed strength'** (- - - - -): adjust c to reproduce **experiment**

⇒ good description of π, ρ, N, Δ ground states

- **'Core model'** ( ): adjust c to reproduce hadronic **quark core w/o pion cloud**

⇒ consistent response in masses, decay constants, charge radii

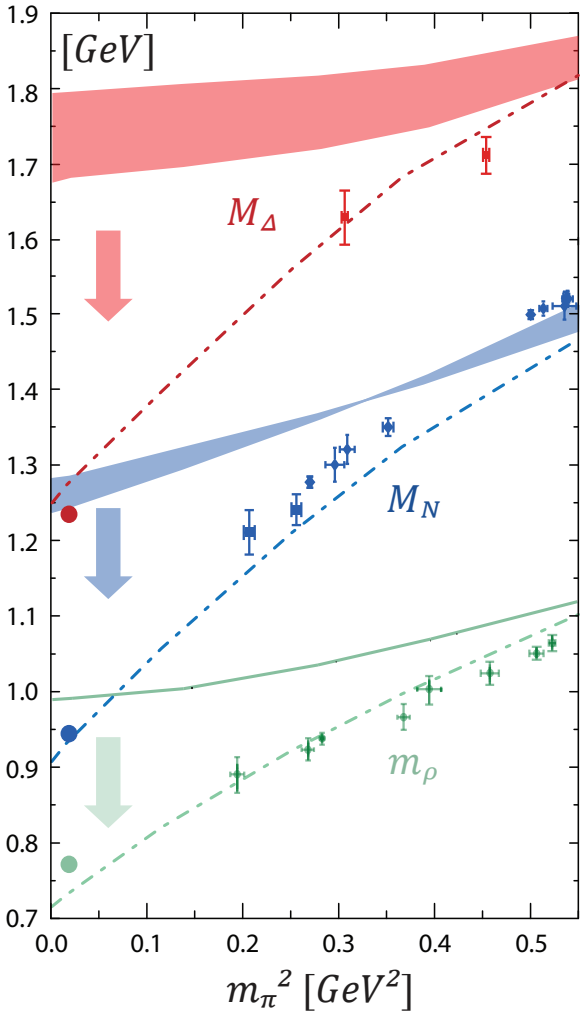




Quark-diquark model

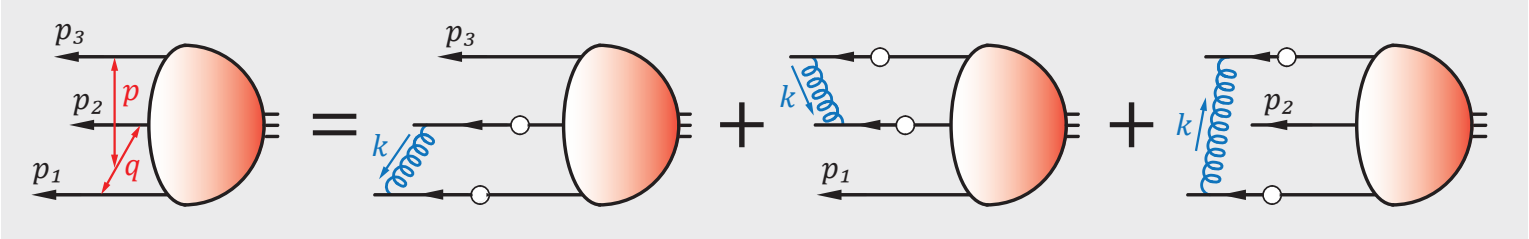
N and Δ masses inherit behavior of meson properties. No baryonic input!

GE *et al.*, PRC 79 & Nicmorus *et al.*, PRD 80



Faddeev equation

in **rainbow-ladder:** any pair of 2 quarks bound by gluon ladder exchange



Nucleon amplitude:

$$\Psi(p,q,P) = \sum_{i=1}^{64} f_i\,(p^2, q^2, p\cdot q, p\cdot P, q\cdot P\,)\;\tau_i\,(p,q,P)$$

64 Dirac covariants

Dominant covariants (→ quark model) :

$$S_{11}^{+} = \Lambda_+(\gamma_5 C)\otimes\Lambda_+ \sim (U^{\uparrow}U^{\downarrow} - U^{\downarrow}U^{\uparrow})\,U^{\uparrow}$$

$$A_{11}^{+} = \Lambda_+(\gamma^{\mu}C)\otimes\gamma^{\mu}\gamma_5\Lambda_+ \sim (U^{\uparrow}U^{\downarrow} + U^{\downarrow}U^{\uparrow})\,U^{\uparrow} - 2\,U^{\uparrow}U^{\uparrow}U^{\downarrow}$$

Faddeev equation

Antisymmetry of the nucleon amplitude under quark exchange:

$$\Psi(p,q,P) = \Big\{ \underbrace{\psi_1(p,q,P)}_{M_A} \underbrace{\text{Flavor}_1}_{M_A} + \underbrace{\psi_2(p,q,P)}_{M_S} \underbrace{\text{Flavor}_2}_{M_S} \Big\} \underbrace{\text{Color}}_{A}$$

	M_A	M_S
Proton:	(ud)u	$\sqrt{2}$ [uu]d − [ud]u
Neutron:	(ud)d	[ud]d − $\sqrt{2}$ [dd]u

For a flavor-independent Faddeev kernel (such as rainbow-ladder):
M_A, M_S **do not mix** ⇒ **2 degenerate solutions** of the equation:

$$M_A \sim S^+_{11}, \ldots \qquad M_S \sim A^+_{11}, \ldots$$

⇒ **what about excitations?**

Faddeev equation

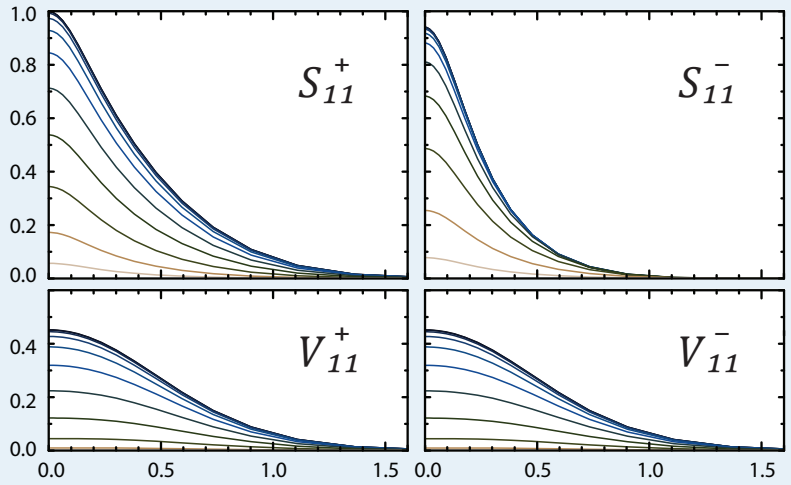
Dominant dressing functions

$f_i\,(p^2, q^2)$

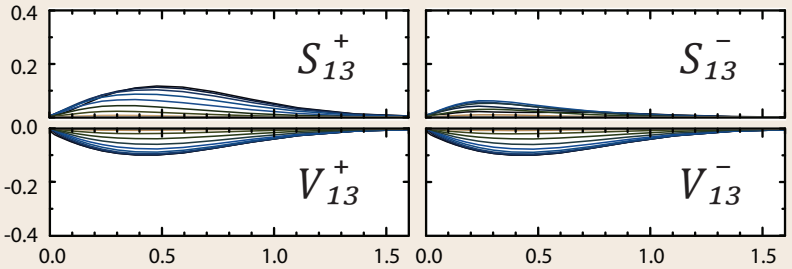
(M_A solution, 0th Chebyshev moments)

$q\ [GeV]$
0.00
0.05
0.09
0.15
0.22
0.32
0.46
0.63
0.86
1.17

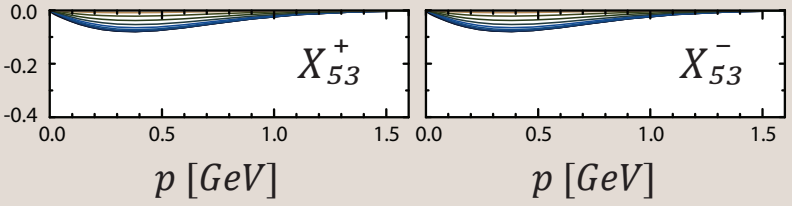
s-wave:
s = 1/2,
l = 0



p-wave:
s = 1/2,
l = 1



p-wave:
s = 3/2,
l = 1



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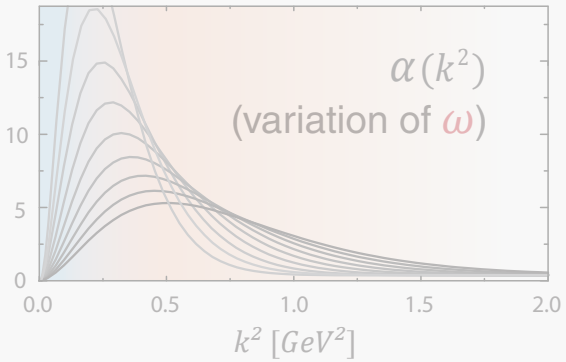

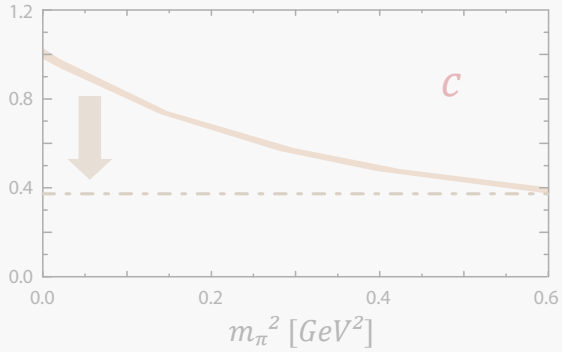

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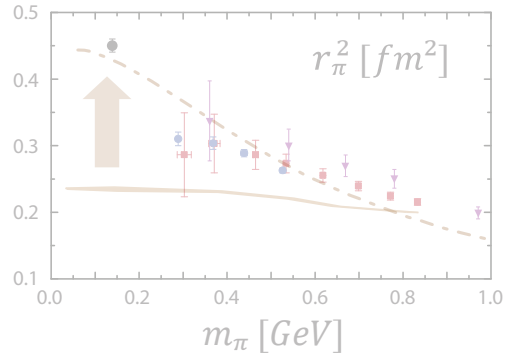
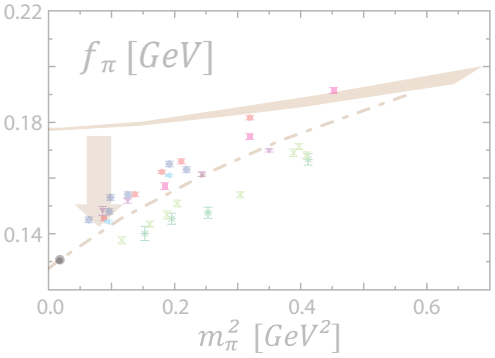
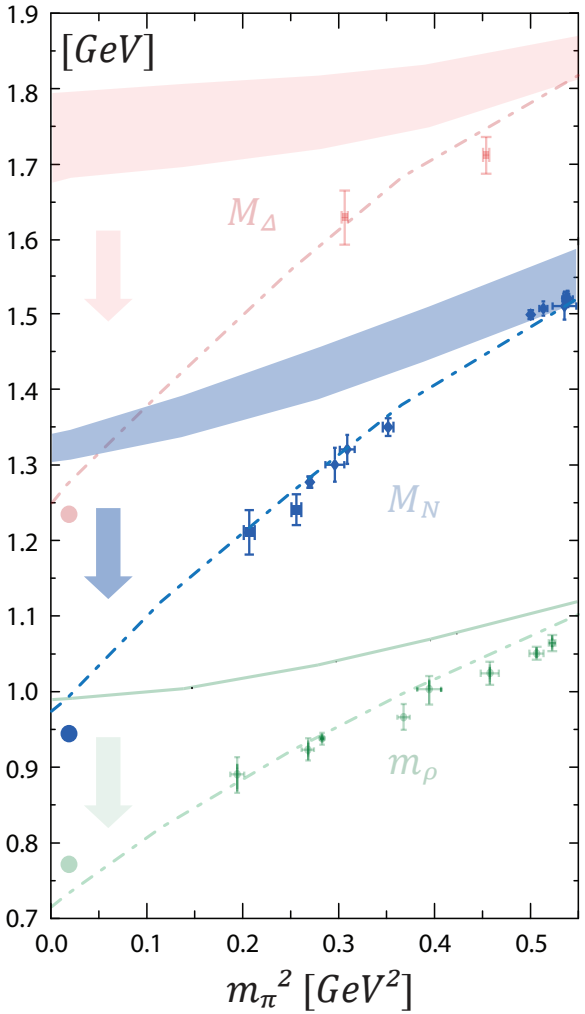




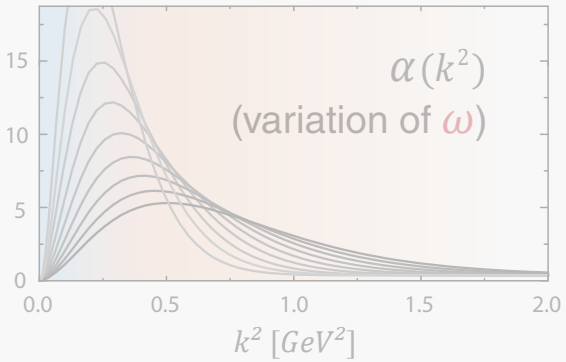
Covariant Faddeev equation
M_N only ~50 MeV larger.
Quark-diquark model works well for the nucleon!

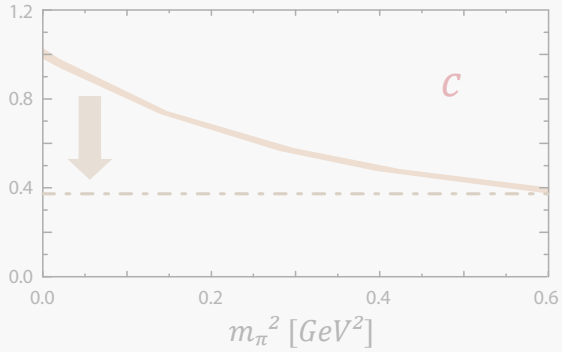


Faddeev equation

Effective coupling:
the only model input!

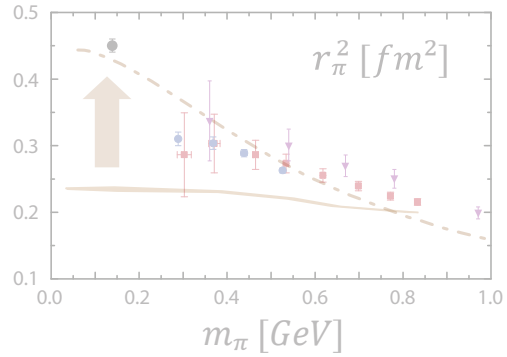
$$\alpha(k^2) = c\,\alpha_{\rm IR}(k^2,\omega) + \alpha_{\rm UV}(k^2)$$

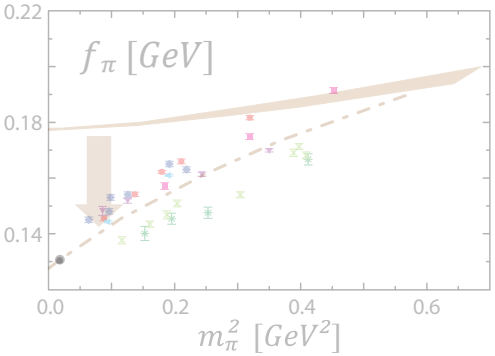




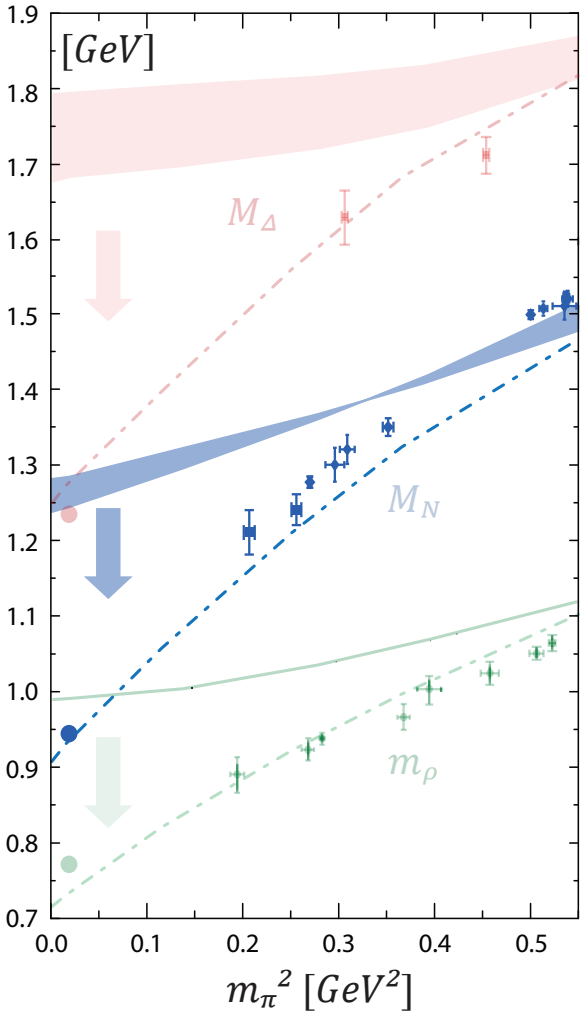
- **'Fixed strength'** (- - - - -):
  adjust c to reproduce **experiment**
  ⇒ good description of π, ρ, N, Δ ground states

- **'Core model'** ( ):
  adjust c to reproduce hadronic **quark core w/o pion cloud**
  ⇒ consistent response in masses, decay constants, charge radii





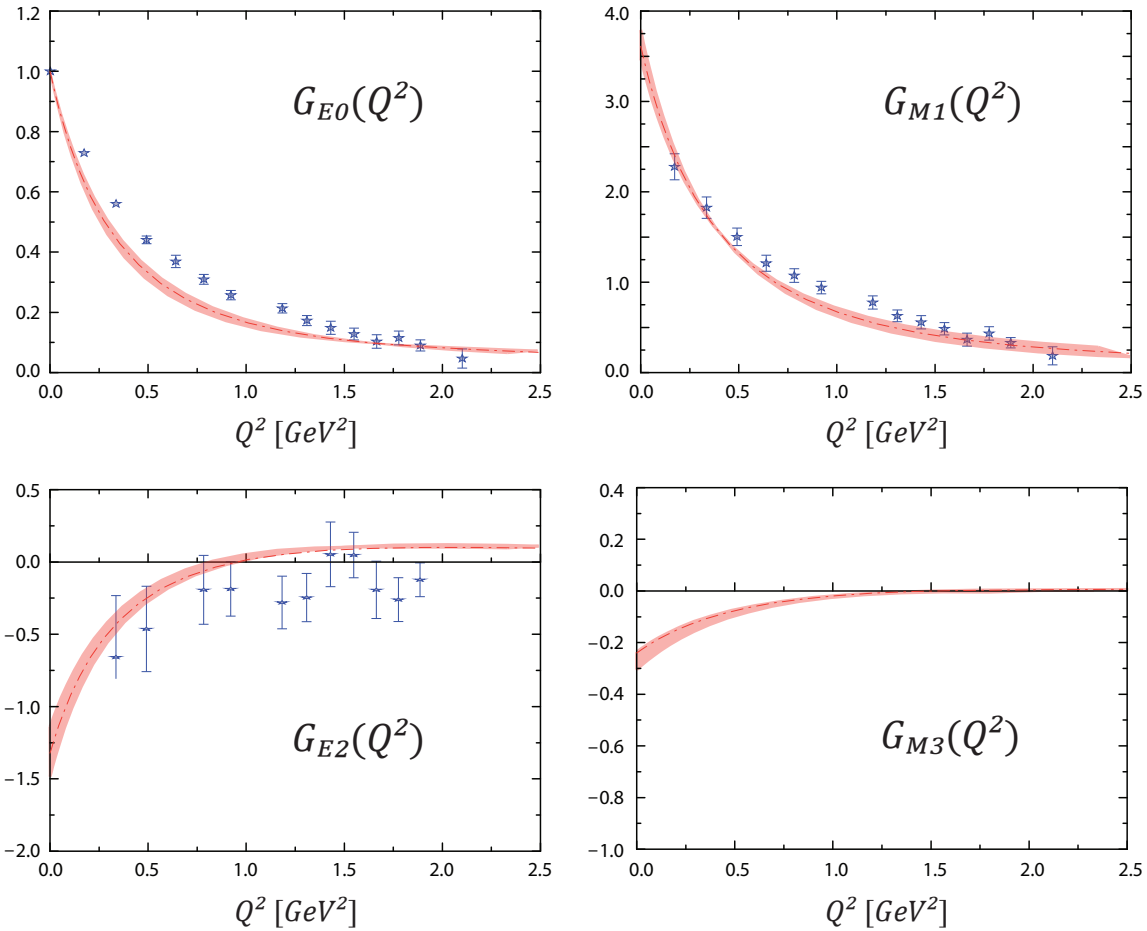
Quark-diquark model
M_N only ~50 MeV larger.
Quark-diquark model works well for the nucleon!



Some recent developments

Deviation between Faddeev equation and quark-diquark model is small.
What about the Δ? We'll know soon. (Even more covariants: 128)

Delta form factors in the quark-diquark model: Nicmorus, GE, Alkofer, in preparation

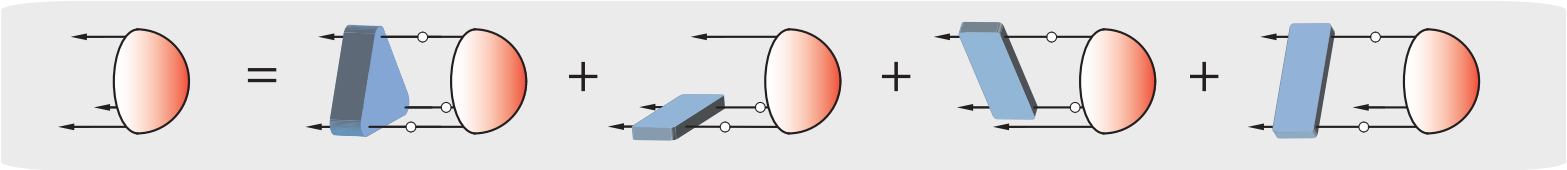


Bands represent variation of width parameter in coupling.

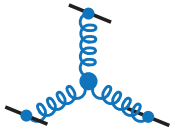
Lattice results:
Alexandrou *et al.*,
PoS LAT2007, 149 (2007)

Summary & Outlook

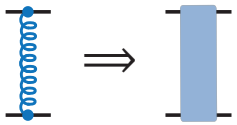
Groundwork for a systematic & covariant description of baryon properties in continuum QCD. Still a lot to be done:



Beyond Faddeev:
Include irreducible 3-body diagrams, e.g.

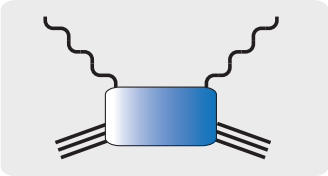


Beyond rainbow-ladder:
must be consistent with AXWTI (chiral symmetry).



Apply to:

- **Excited baryons**
- **Nucleon, Delta, N→$\Delta\gamma$ transition form factors**
  ↳ in particular at large Q^2
- $NN\gamma\gamma$ **vertex:**



- **Two-photon corrections** to proton's form factor ratio
- **DVCS,** generalized parton distributions, nucleon structure
- **Proton-antiproton physics** @ FAIR

Thanks for your attention.

⇒ GE, R. Alkofer, A. Krassnigg, D. Nicmorus:

- PRL 104, 201601 (2010) (0912.2246)
- EPJ Web Conf. 3, 3028 (2010) (0912.2876)